0:00 / 3:13

Mixed designs and the GLM

Professor Andy Field

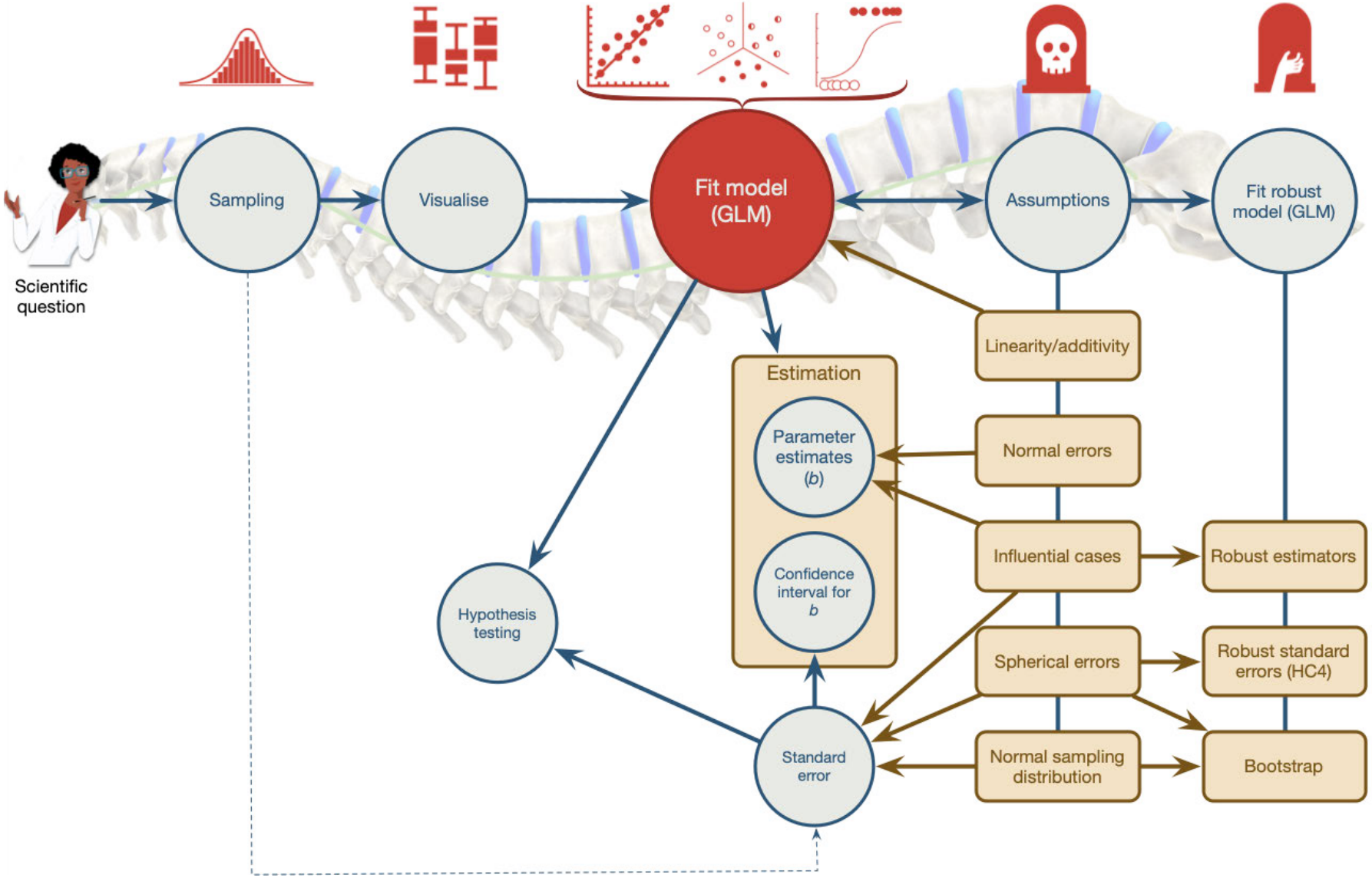
@profandyfield

www.youtube.com/user/ProfAndyField/

www.discoveringstatistics.com

www.milton-the-cat.rocks

www.discovr.rocks

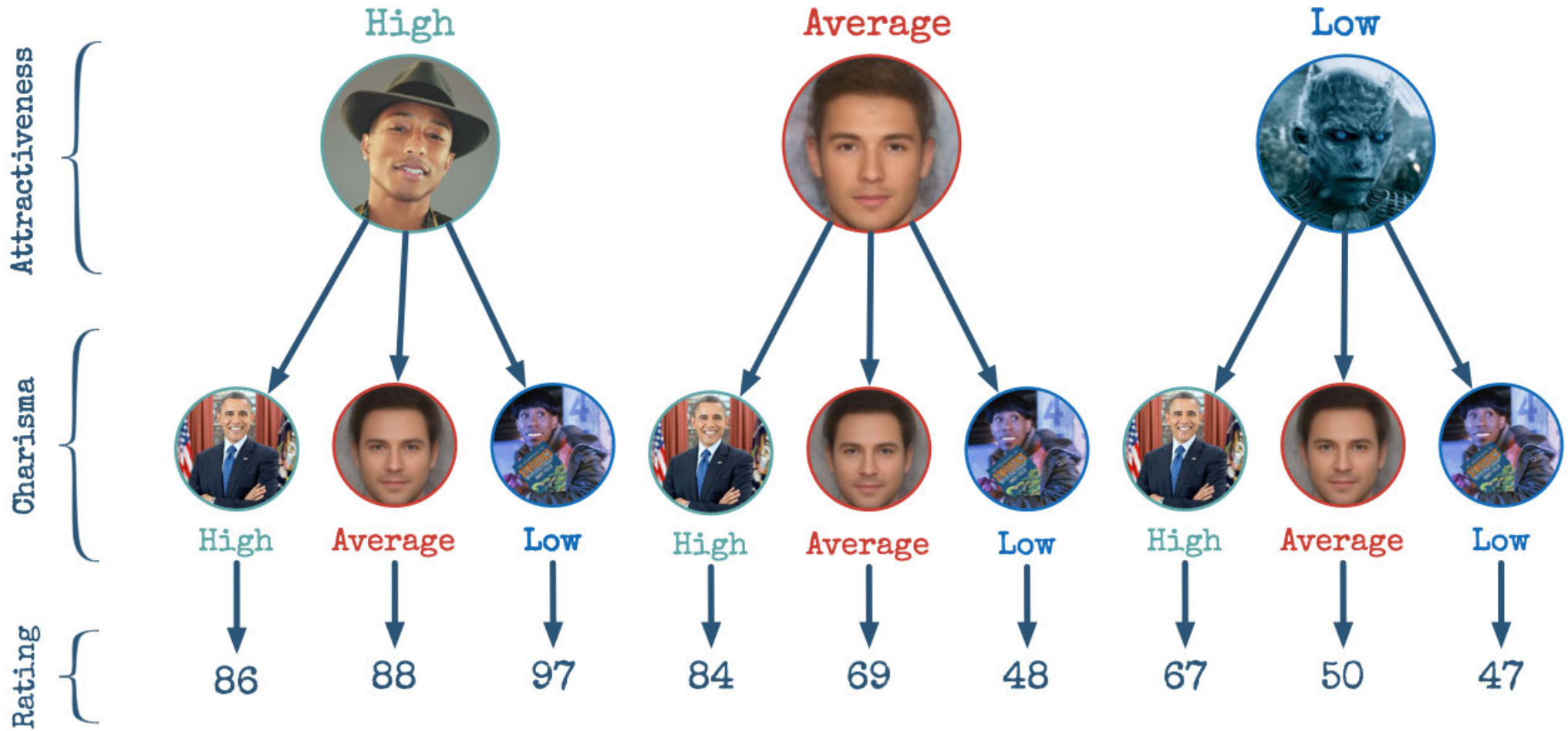

Scientific question
Sampling
Visualise
Fit model (GLM)
Assumptions
Fit robust model (GLM)
Linearity/additivity
Estimation
Parameter estimates (b)
Confidence interval for b
Normal errors
Influential cases
Robust estimators
Hypothesis testing
Spherical errors
Robust standard errors (HC4)
Standard error
Normal sampling distribution
Bootstrap

Learning outcomes

- Describe what is meant by a mixed design
- Apply a GLM to a mixed designs using `afex`
- Interpret a GLM for a mixed design (up to three factors)
- Making sense of three-way interactions

A speed-dating example

- Dating strategies
  - If someone is committed to pursuing a relationship with a person who plays hard to get, they will find that person more desirable but less likeable (Dai, Dong, and Jia , 2014)
  - Field (2017) fictitious study to look at the interplay between looks, personality and dating strategies on evaluations of a date
- Design
  - Predictor 1 (looks): above average (high), average, below average (low)
  - Predictor 2 (personality): high charisma, average charisma, low charisma
  - Predictor 3 (strategy): acting normal or playing hard to get
- Outcome
  - P's percentage desire for a full date (0-100%)


Attractiveness
Charisma
Rating
High
Average
Low
High
Average
Low
High
Average
Low
High
Average
Low
86
88
97
84
69
48
67
50
47

The data in R

Table 1: Data for the speed dating example

	id	strategy	looks	personality	date
1	2rl3f	Hard to get	High	High	86
2	2rl3f	Hard to get	Average	High	84
3	2rl3f	Hard to get	Low	High	67
4	2rl3f	Hard to get	High	Average	88
5	2rl3f	Hard to get	Average	Average	69
6	2rl3f	Hard to get	Low	Average	50
7	2rl3f	Hard to get	High	Low	97
8	2rl3f	Hard to get	Average	Low	48
9	2rl3f	Hard to get	Low	Low	47
10	2h53u	Hard to get	High	High	91

Mixed designs and the linear model

- A bare minimum
  - Modelling variation in ratings overall across participants
  - Not modelling variation in the effect of repeated measures predictors across participants
  - Not explicitly listing the dummy variables for looks and personality (they'll be transformed into two dummy variables each)

$$\hat{\text{date}}_{ij} = \hat{b}_{0j} + \hat{b}_1\text{strategy}_i + \hat{b}_2\text{looks}_{ij} + \hat{b}_3\text{personality}_{ij} + \ldots$$
$$\cdots + \hat{b}_4\text{strategy}_i \times \text{looks}_{ij} + \hat{b}_5\text{strategy}_i \times \text{personality}_{ij} + \hat{b}_6\text{looks}_{ij} \times \text{personality}_{ij} + \ldots$$
$$\cdots + \hat{7}_5\text{strategy}_i \times \text{looks}_{ij} \times \text{personality}_{ij} + e_{ij} + \hat{u}_{0j}$$

Summary of effects

We will get an *F*-statistic for the following effects:

- Main effects
  - looks
  - personality
  - strategy
- Two-way interactions
  - strategy × looks
  - strategy × personality
  - looks × personality
- Three-way Interaction
  - strategy × looks × personality

Repeat the following mantras:

"It is never sensible to interpret main effects in the presence of a significant interaction effect."

"It is not sensible to interpret interaction effects in the presence of a significant higher-order interaction effect."

Fitting the model

- The `afex::aov_4()` function
  - Specify the repeated measures with `(rm_predictors|id_var)`
  - Automatically sets contrasts
  - Built in interaction plot with `afex_plot()`
  - But … no parameter estimates, diagnostic plots, or robust methods

```
date_afx <- afex::aov_4(
  date ~ strategy*looks*personality +
(looks*personality|id),
  data = sniff_tib
  )
```

Contrasts

- For both looks and personality there are three categories (high, average, low). These variables need to be coded into two dummy/contrast variables.
- We'd expect the desirability of a full date to be different at the extremes to the 'average'. Therefore, for both predictors these contrasts would work:
    - Contrast 1: {high} vs. {average}
    - Contrast 2: {low} vs. {average}
- We can extract these using `emmeans::contrast()`
- Selection of methods:
    - `eff`: each category is compared to the average of all categories (default).
    - `pairwise`: each category is compared to all others.
    - `poly`: polynomial contrasts (i.e. tests for an ordered trend in category means).
    - `trt.vs.ctrl`: compares each category to a declared reference category (by default the first category). Use `ref = x` to make `x` the reference category.
    - `consec`: compares each level/category (except the first) to the previous

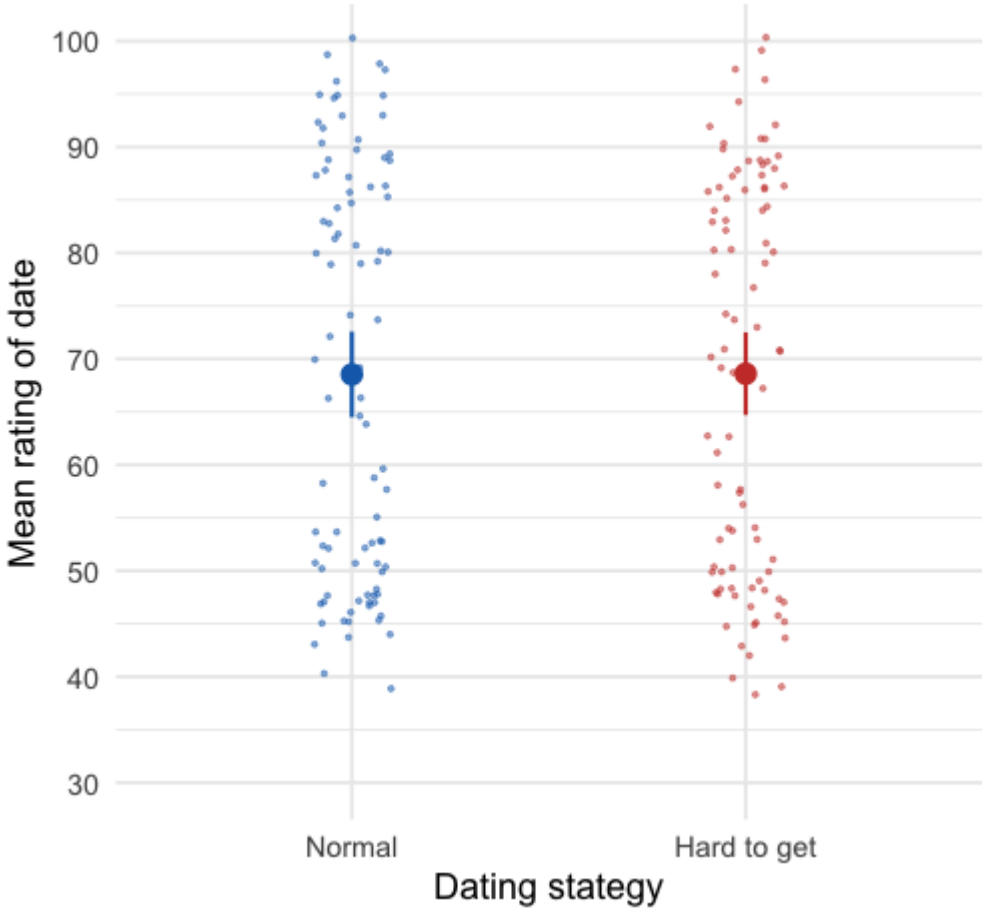
Overall model summary

```
afex::aov_4(date ~ strategy*looks*personality + (looks*personality|id), data = date_tib)
```

	num Df	den Df	MSE	F	ges	Pr(>F)
strategy	1.00	18.00	42.23	0.00	0.00	0.95
looks	1.92	34.62	25.50	423.73	0.81	0.00
strategy:looks	1.92	34.62	25.50	80.43	0.45	0.00
personality	1.87	33.62	37.89	328.25	0.83	0.00
strategy:personality	1.87	33.62	37.89	62.45	0.47	0.00
looks:personality	3.20	57.55	34.62	36.63	0.45	0.00
strategy:looks:personality	3.20	57.55	34.62	24.12	0.35	0.00

✎ The strategy × looks × personality interaction was significant, $F(3.20, 57.55) = 24.12$, $p < 0.001$. The effect size suggested that this interaction accounted for about a third of the available variance in date ratings, $\eta^2_G = 0.35$.

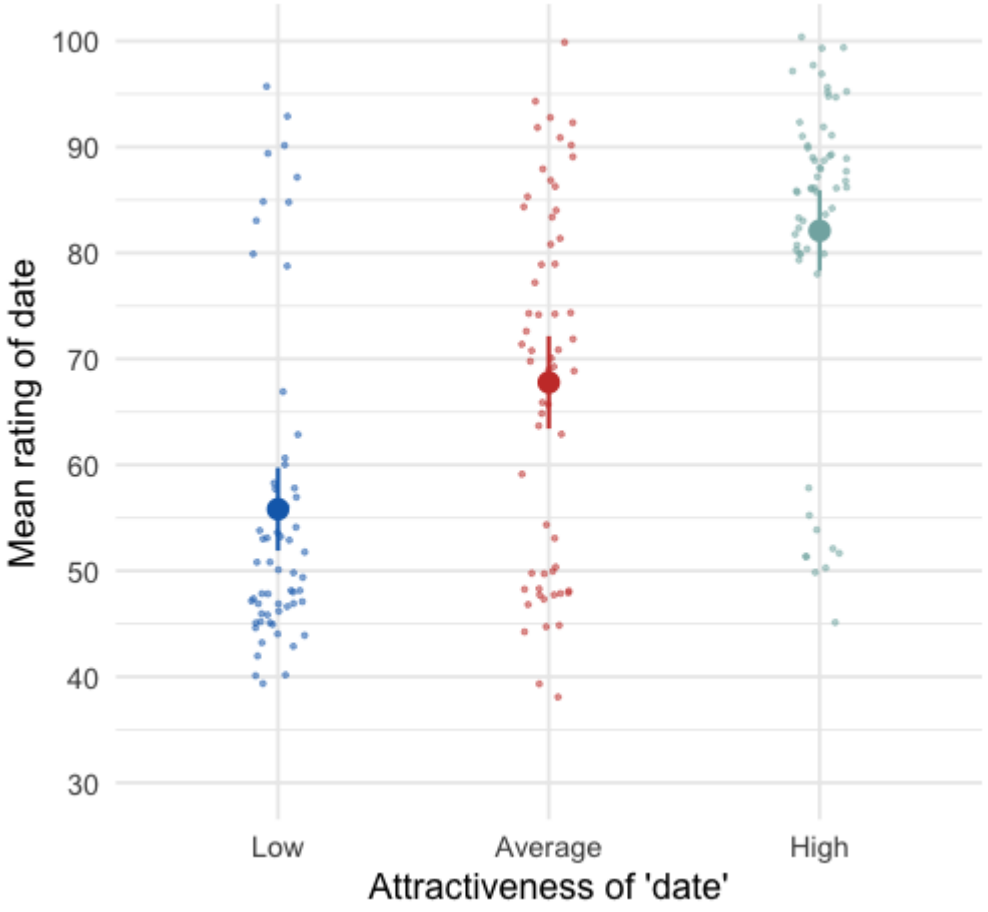
Main effect of strategy




```
emmeans::emmeans(date_afx, ~strategy, model = "multivariate")
```

strategy	emmean	SE	df	lower.CL	upper.CL
Normal	68.53	0.69	18	67.09	69.97
Hard to get	68.60	0.69	18	67.16	70.04

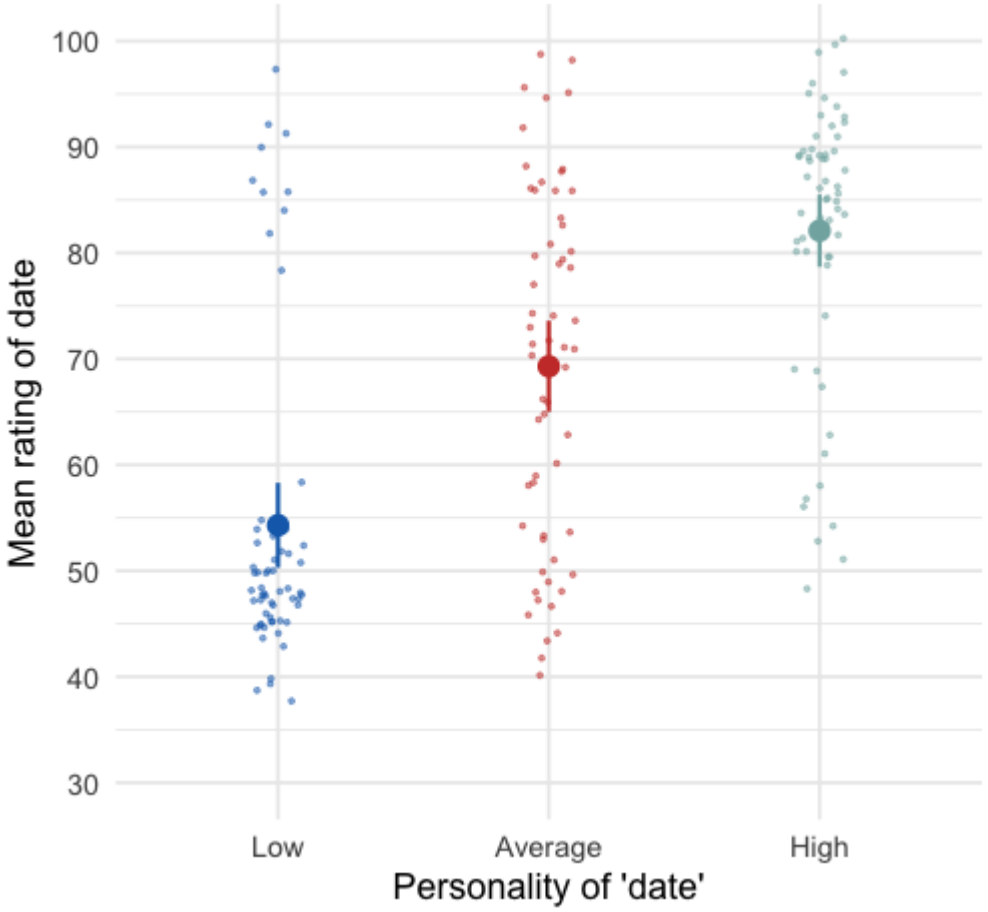
Main effect of looks




```
emmeans::emmeans(date_afx, ~looks, model =
"multivariate")
```

looks	emmean	SE	df	lower.CL	upper.CL
Low	55.82	0.65	18	54.45	57.18
Average	67.78	0.82	18	66.06	69.51
High	82.10	0.65	18	80.73	83.47

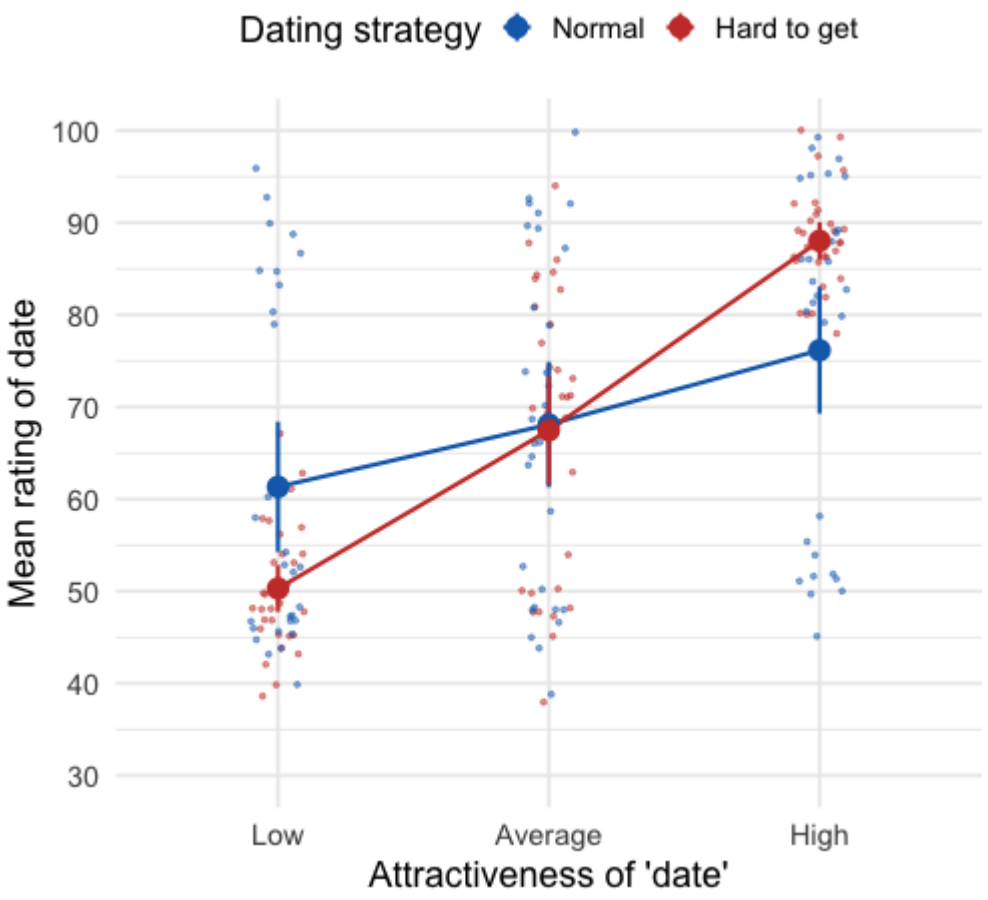
Main effect of personality




```
emmeans::emmeans(date_afx, ~personality,
model = "multivariate")
```

personality	emmean	SE	df	lower.CL	upper.CL
Low	54.3	0.57	18	53.10	55.50
Average	69.3	0.73	18	67.76	70.84
High	82.1	1.01	18	79.98	84.22

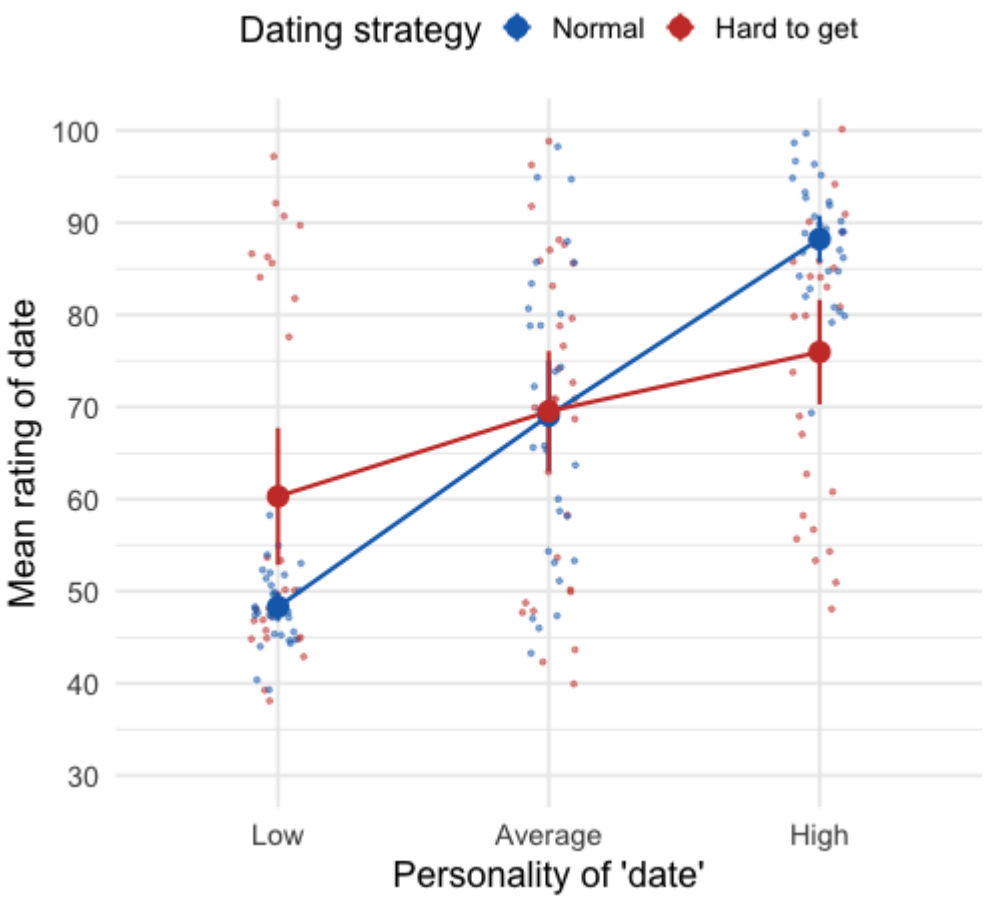
Strategy × looks interaction




```
emmeans::emmeans(date_afx, c("strategy",
"looks"), model = "multivariate")
```

strategy	looks	emmean	SE	df	lower.CL	upper.CL
Normal	Low	61.33	0.92	18	59.40	63.27
Hard to get	Low	50.30	0.92	18	48.37	52.23
Normal	Average	68.10	1.16	18	65.66	70.54
Hard to get	Average	67.47	1.16	18	65.03	69.90
Normal	High	76.17	0.92	18	74.23	78.11
Hard to get	High	88.03	0.92	18	86.09	89.97

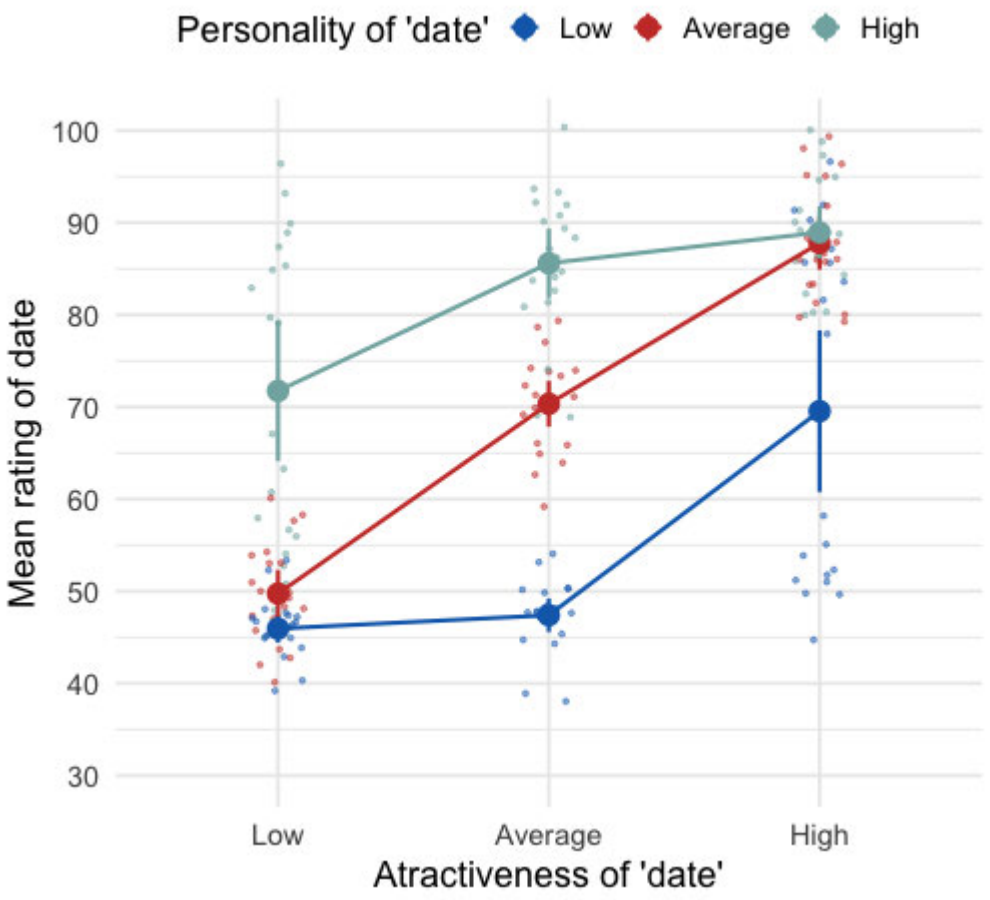
Strategy × personality interaction




```
emmeans::emmeans(date_afx, c("strategy",
"personality"), model = "multivariate")
```

strategy	personality	emmean	SE	df	lower.CL	upper.CL
Normal	Low	48.30	0.81	18	46.60	50.00
Hard to get	Low	60.30	0.81	18	58.60	62.00
Normal	Average	69.07	1.03	18	66.89	71.24
Hard to get	Average	69.53	1.03	18	67.36	71.71
Normal	High	88.23	1.43	18	85.23	91.23
Hard to get	High	75.97	1.43	18	72.97	78.97

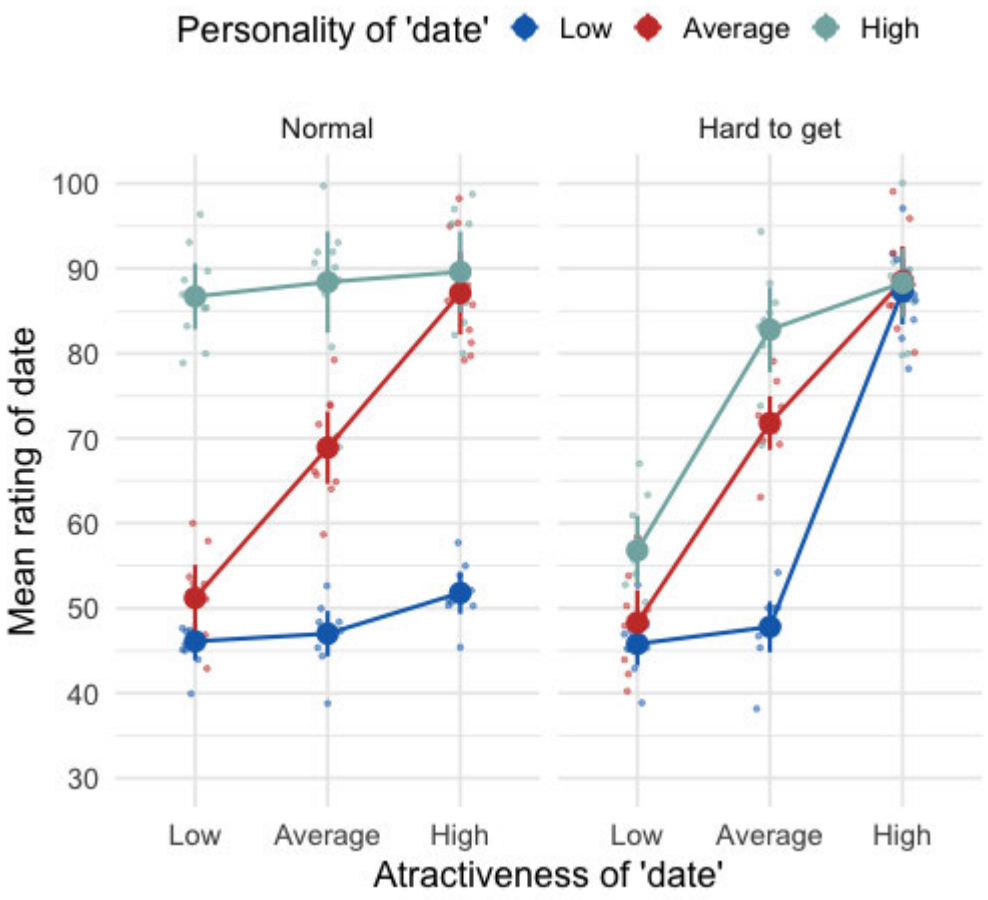
Looks × personality interaction




```
emmeans::emmeans(date_afx, c("looks",
"personality"), model = "multivariate")
```

looks	personality	emmean	SE	df	lower.CL	upper.CL
Low	Low	45.95	0.75	18	44.38	47.52
Average	Low	47.40	0.89	18	45.54	49.26
High	Low	69.55	1.02	18	67.41	71.69
Low	Average	49.75	1.21	18	47.21	52.29
Average	Average	70.35	1.17	18	67.89	72.81
High	Average	87.80	1.41	18	84.84	90.76
Low	High	71.75	1.25	18	69.13	74.37
Average	High	85.60	1.72	18	81.98	89.22
High	High	88.95	1.38	18	86.04	91.86

Strategy × looks × personality interaction



strategy	looks	personality	emmean	SE	df	lower.CL	upper.CL
Normal	Low	Low	46.1	1.06	18	43.88	48.32
Hard to get	Low	Low	45.8	1.06	18	43.58	48.02
Normal	Average	Low	47.0	1.26	18	44.36	49.64
Hard to get	Average	Low	47.8	1.26	18	45.16	50.44
Normal	High	Low	51.8	1.44	18	48.77	54.83
Hard to get	High	Low	87.3	1.44	18	84.27	90.33
Normal	Low	Average	51.2	1.71	18	47.60	54.80
Hard to get	Low	Average	48.3	1.71	18	44.70	51.90
Normal	Average	Average	68.9	1.66	18	65.42	72.38
Hard to get	Average	Average	71.8	1.66	18	68.32	75.28
Normal	High	Average	87.1	1.99	18	82.92	91.28
Hard to get	High	Average	88.5	1.99	18	84.32	92.68
Normal	Low	High	86.7	1.77	18	82.99	90.41
Hard to get	Low	High	56.8	1.77	18	53.09	60.51
Normal	Average	High	88.4	2.43	18	83.29	93.51
Hard to get	Average	High	82.8	2.43	18	77.69	87.91
Normal	High	High	89.6	1.96	18	85.49	93.71
Hard to get	High	High	88.3	1.96	18	84.19	92.41

Contrasts across the interaction

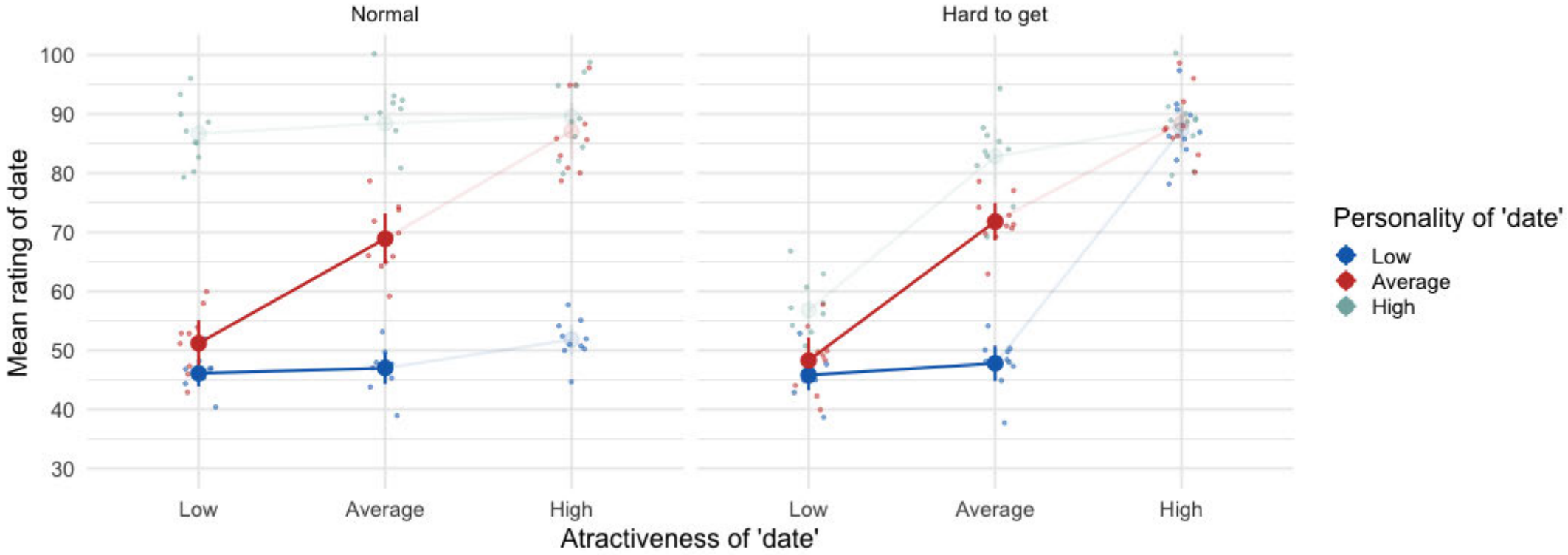
```
three_way_emm <- emmeans::emmeans(date_afx, c("strategy", "looks", "personality"), model =
"multivariate")

emmeans::contrast(
  three_way_emm,
  interaction = c(strategy = "trt.vs.ctrl",
                  looks = "trt.vs.ctrl",
                  personality = "trt.vs.ctrl"),
  ref = 2,
  adjust = "holm"
  )
```

strategy_trt.vs.ctrl	looks_trt.vs.ctrl	personality_trt.vs.ctrl	estimate	SE	df	t.ratio	p.value
Normal - Hard to get	Low - Average	Low - Average	-4.7	4.07	18	-1.16	0.53
Normal - Hard to get	High - Average	Low - Average	-36.2	4.65	18	-7.79	0.00
Normal - Hard to get	Low - Average	High - Average	18.5	5.41	18	3.42	0.01
Normal - Hard to get	High - Average	High - Average	-5.8	6.02	18	-0.96	0.53

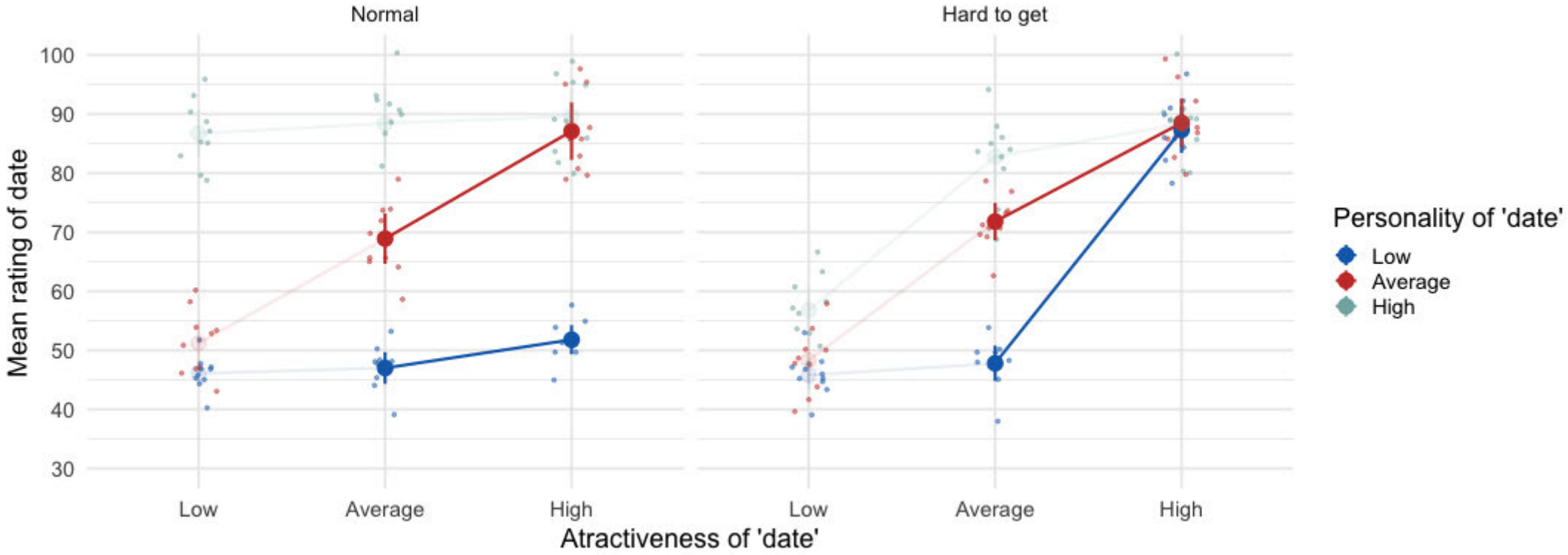
Contrast 1

strategy_trt.vs.ctrl	looks_trt.vs.ctrl	personality_trt.vs.ctrl	estimate	SE	df	t.ratio	p.value
Normal - Hard to get	Low - Average	Low - Average	-4.7	4.07	18	-1.16	0.26



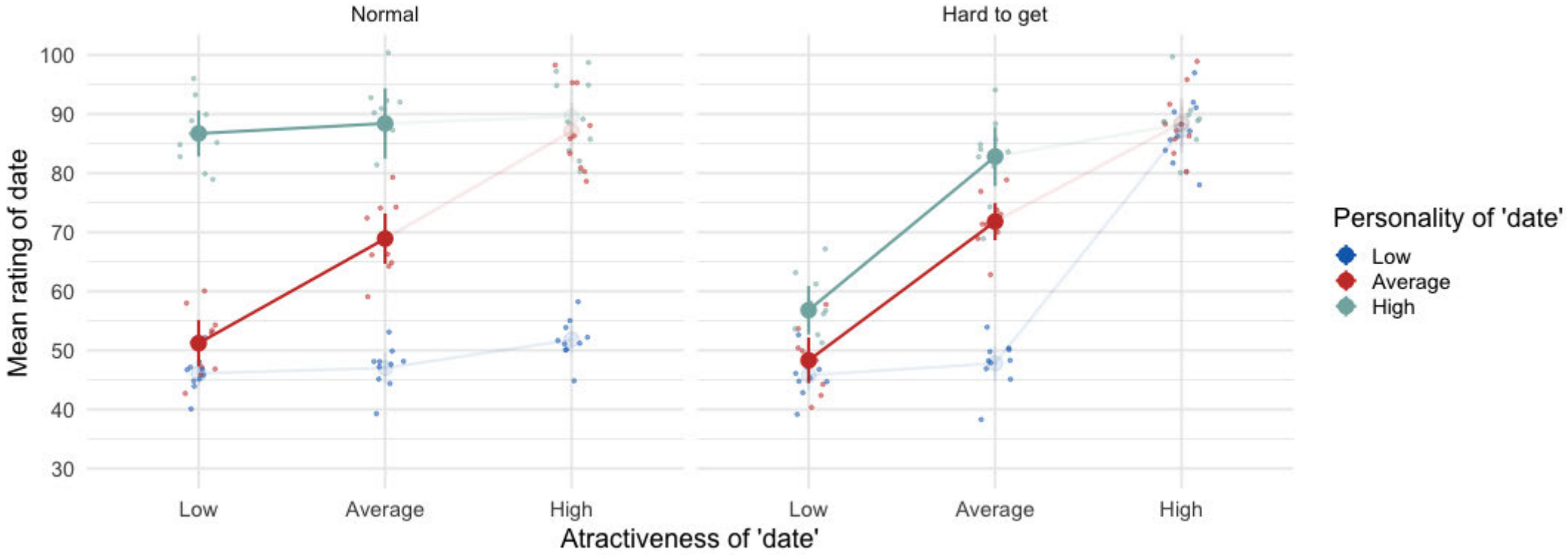
Contrast 2

	strategy_trt.vs.ctrl	looks_trt.vs.ctrl	personality_trt.vs.ctrl	estimate	SE	df	t.ratio	p.value
2	Normal - Hard to get	High - Average	Low - Average	-36.2	4.65	18	-7.79	0



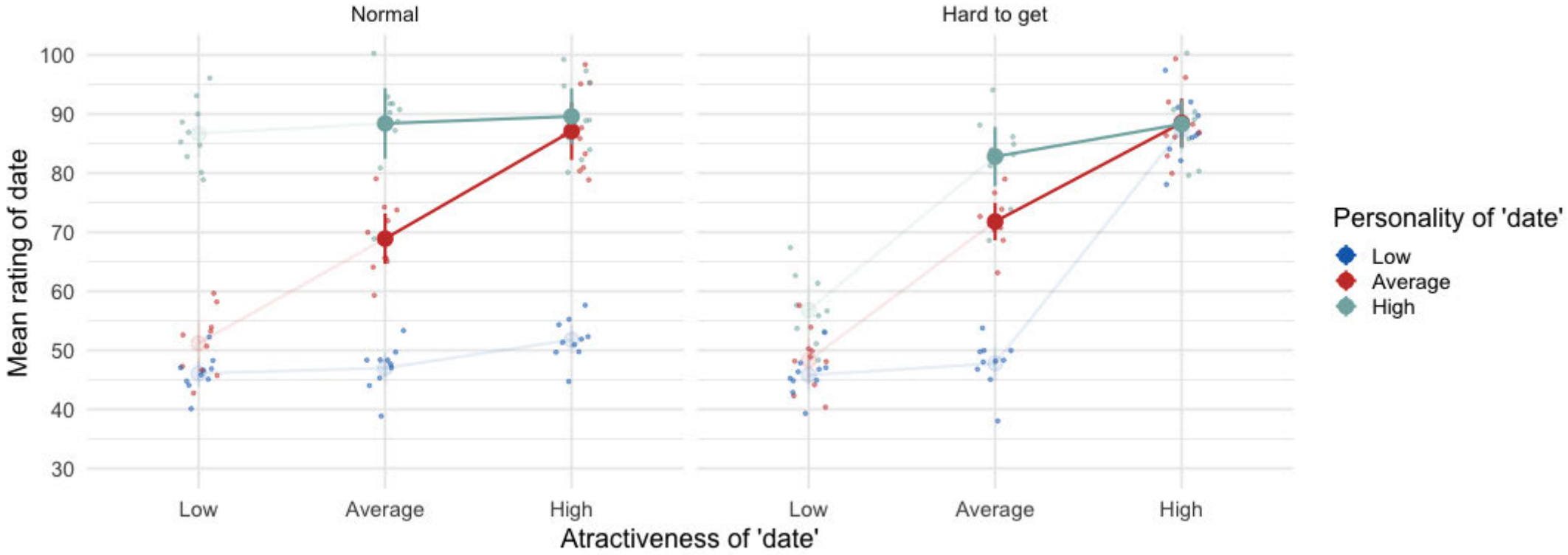
Contrast 3

	strategy_trt.vs.ctrl	looks_trt.vs.ctrl	personality_trt.vs.ctrl	estimate	SE	df	t.ratio	p.value
3	Normal - Hard to get	Low - Average	High - Average	18.5	5.41	18	3.42	0



Contrast 4

	strategy_trt.vs.ctrl	looks_trt.vs.ctrl	personality_trt.vs.ctrl	estimate	SE	df	t.ratio	p.value
4	Normal - Hard to get	High - Average	High - Average	-5.8	6.02	18	-0.96	0.35



Summary

- Mixed designs combine at least one predictor measured using repeated measures, and at least one measured using different entities
- Sphericity
    - Correct using Greenhouse-Geisser, report it and forget about it
    - `afex` does this by default
- The output is much the same as an other designs
    - An *F*-statistic for each effect
    - Interpret as you would any *F*-statistic
    - Pick apart the highest order interaction
- Interactions
    - Use built in contrasts to tease apart what the interaction shows